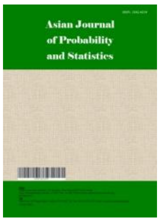

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A New Class of Life Distributions Based on Moment Inequalities

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper, a new class of life distribution is named new better than renewal used in moment generating function ($NBRU_{mgf}$). A new test for exponentiality versus $NBRU_{mgf}$ based on moment inequalities is established. Pitman's asymptotic efficiencies, powers and critical values of the new test are calculated to assess the performance of the test. The right censored data is handled also. Finally some applications are applied to the new test.

Keywords: $NBRU_{mgf}$; exponential; pitman's asymptotic efficiencies; powers; censored data.

1 Introduction

Aging notions have been the subject of investigation for more than three decades and have played an important role in reliability theory. Concepts of aging describe how a population of units or systems can be improved or deteriorated with age. We study some statistical properties and probabilistic distributions, then some classes of life distributions describing aging. Some classes of life distribution which have been introduced are based on new better than used NBU Barlow & Proschan [1] and new better than renewal used (*NBRU*), new better than used in increasing concare order (NBU) Desphpan, et al., [2].

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2 Motivation

2.1 Renewal survival function

Consider a component with life time X with distribution function F(x), is put in operation. When the failure occurs, the component will be replaced by a sequence of mutually and identically components which are independent of the first component. In the long time, the remaining life distribution of a component in operation at time (t) is called the stationary renewal distribution. The corresponding renewal survival function is:

$$\overline{W} = \frac{1}{\mu}\int_{0}^{\infty}\overline{F(u)du}$$

Testing exponentially versus some kinds of classes such as NBU, NBRU, among others to know others we refer the researches Ahmed [3], Al Wasel et al., [4], Mahmoud et al., [5] and Ahmed and Mugdadi [6].

A non negative random variable X is said to be new better than renewal used in moment generating function (denoted by ($NBRU_{mgf}$) if, and only, if:

$$\int_{0}^{\infty} e^{sx}\overline{W}\,(x+t)dx \le \overline{W}(t)\int_{0}^{\infty} e^{sx}\overline{F}(x)dx, \quad s \ge 0$$

The $NBRU_{mgf}$ class expanding the NBRU classes, in general to know the true value of population parameters, we estimate hypothesis test which is given the symbol H_0: null hypothesis and symbol H_1: the alternative hypothesis, when the null hypothesis is false, the alternative hypothesis is true in which both the null and alternative hypothesis should be started before any statistical test of significance is conducted.

In this paper, we developed moment inequalities of the $NBRU_{mgf}$ class then we needed to test H_0: F exponential distribution against alternatives that H_1: F is $NBRU_{mgf}$ and non exponential is presented. We found that the test statistic is simple and we calculated pitman asymptotic efficiency. We calculated critical values for sample size n = 5 (5) 50. We estimated the power also. We developed testing for censored data.

Finally, we discussed some applications to elucidate the usefulness of the proposed test in reliability analysis for $(NBRU)_{mgf}$ distributions.

3 Moment Inequalities

In the following theorem, the moment inequalities for the $NBRU_{mgf}$ class are derived and assumed that all moments are exist and finite.

2.1 Theorem

If F is $NBRU_{mgf}$, then for all integer r ≥ 0

$$\frac{\mu_{r+2}(\phi(s)-1)}{s\mu(r+1)(r+2)} \ge \frac{r!}{s^{r+1}}\left[\frac{(\phi(s)-1)}{s^2\mu} - \frac{1}{s}\right] - \frac{r!}{s^{r+1}}\sum_{1=0}^{r}\frac{s^i}{i!}\,\frac{\mu_{i+2}}{i!\,(i+1)(i+2)\mu} \qquad (2.1)$$

Where

$$\phi(s) = \int_{0}^{\infty} e_{sx}\, dF(x)$$

Proof:

Since F is $NBRU_{mgf}$, then:

$$\int_0^\infty t^r\,\overline{W}(t)dt\int_0^\infty e^{sx}\overline{F}(x)dx \geq \int_0^\infty\int_0^\infty t^r\,e^{sx}\,\overline{W}(x+t)dxdt \tag{2.2}$$

Note that:

$$\int_0^\infty t^r\,\overline{W}(t)dt = E\left[\int_0^\infty t^r(X-t)I(X>t)dt\right] = \frac{\mu_{r+2}}{\mu(r+1)(r+2)}$$

It is easy to show that:

$$\int_0^\infty e^{sx}\overline{F}(x)dx = E\int_0^\infty I(X>x)e^{sx}\,dx = \frac{1}{s}(\phi(s)-1)$$

Then the left hand side of Eq. (2.2) is:

$$\frac{\mu_{r+2}(\phi(s)-1)}{s\mu\mu(+1)(r+2)} \tag{2.3}$$

Let u = x + t, then du = dx and v = t then dv = dt,

So the right hand side of Eq. (2.2) is:

$$\int_0^\infty e^{sv}\overline{W}(v)\int_0^v u^r\,e^{-su}\,dudv$$

Since:

$$\int_0^v u^r\,e^{-su}\,du = \frac{r!}{s^{r+1}}\left[1-\sum_{i=0}^{r}\frac{(sv)^i}{i!}e^{-sv}\right],\ v>0 \text{ (Incomplete gamma function)}$$

Then the right hand side of Eq. (2.1):

$$\frac{r!}{s^{r+1}}\left[\frac{(\phi(s)-1)}{s^2\mu}-\frac{1}{s}\right]-\frac{r!}{s^{r+1}}\sum_{I=0}^{r}\frac{s^i}{i!}\frac{\mu_{i+2}}{(i+1)(i+2)\mu} \tag{2.4}$$

Making use of (2.2) - (2.4), the result follows.

When r = 1 Eq. 2.1 reduces to:

$$\frac{\mu_3(\phi(s)-1)}{6s\mu} \geq \frac{1}{s^2}\left[\left(\frac{(\phi(s)-1)}{s^2\mu}-\frac{1}{s}\right)-\left(\frac{\mu_2}{2\mu}+\frac{s\mu_3}{6\mu}\right)\right]$$

4 Testing Exponential Versus *NBRU$_{mgf}$* Class

In view of theorem 2.1 with r ≥ 0, we develop a test H_0: F is exponential against an alternative that H_1: F is $NBRU_{mgf}$. We may use δ as a measure of departure from exponential where:

$$\delta = \frac{\mu_{r+1}}{s(r+1)(r+2)}(\phi(s)-1) - \frac{r!}{s^{r+1}}\left[\frac{(\phi(s)-1)}{s^2\mu} - \frac{1}{s}\right] + \frac{r!}{s^{r+1}}\sum_{i=0}^{r}\frac{i^i}{i!}\frac{\mu_{i+2}}{(i+1)(i+2)\mu}$$

If r = 1, then δ(s) reduce to:

$$\delta = \frac{6s\mu + 3s^2\mu_2 + 6 + (s^3\mu_3 - 6)\phi(s)}{s^3}. \tag{3.1}$$

Note that under H_1 : $\delta = 0$, while under H_1 : $\delta > 0$.

To estimate δ, let $X_1, X_2, \ldots, X_n$ be a random sample from *F*. So the empirical form of δ in Eq. (3.1) is:

$$\delta_n = \frac{1}{n^2 s^3}\sum_{i=1}^{n}\sum_{j=0}^{n}\left[6sX_i + 3s^2 X_i^2 + 6 + (s^3 X_i^3 - 6)\, e^{sx_i}\right]$$

Theorem 2.2

(1) As $n \to \sqrt{n}\,(\hat{\delta}_n - \delta)$ is asymptotically normal with zero mean and variance σ^2, where σ^2 is given by:

$$\sigma^2 = Var\left[3s^2 X_1^2 + 6sX_1 + \frac{\left(s^3 X_1^3 - 6\right)}{1-s} + 6\,(s^3 - 6)\, e^{sx_1} + 6s^2 + 6s + 12\right] \tag{3.2}$$

(2) Under H_0, the variance σ_0^2 is:

$$\frac{-36s^8(19 + (s-14)s)}{(s-1)^4(2s-1)}, \; s < \frac{1}{2}$$

Proof

Set

$$\psi(X_1, X_2) = 6sX_i + 3s^2 X_i^2 + 6 + (s^3 X_i^2 - 6)\, e^{sX_i}$$

Using U-statistic theory (see Lee, (1990)), the variance can be found as follows:

$$\sigma^2 = Var(\phi(X))$$

Where:

$$\phi(X) = \phi_1(X) + \phi_2(X),$$
$$\phi_1(X) = E[\psi(X_1, X_2)\, X_1]$$

$$= 3s^2 X_1^2 + 6sX_1 + 6 + \frac{(s^3 X_1^3 - 6)}{1-s}$$

And:

$$\phi^2 (X) = E [\psi (X_1 , X_2) X_2]$$

$$= 6s^2 + 6s + 6 + 6 (s^3 - 1) e^{sX1}$$

So,

$$\phi(X) = 3s^2 X_i^2 + 6sX_1 + \frac{(s^3 X_1^3 - 6)}{1-s} + 6 (s^3 - 6) e^{sx1} + 6 s + 12$$

And Eq. (3.2) is deduced.

Under H_0 , it is easy to prove that $\mu_0 = E(\phi (X)) = 0$, and the variance σ_0^2 reduce to:

$$\frac{-36s^8 (19 + (s-14)s)}{(s-1)^4 (2s-1)} \quad , \quad s < \frac{1}{2}$$

5 Pitman Asymptotic Efficiency

In this section, the Pitman asymptotic efficiencies (PAEs) of our test δ_n are computed for the following alternatives:

(1) Linear failure rate family (LFR):

$$\overline{F_1} (x) = e^{-x - \frac{\theta}{2} x^2} \quad , x \geq 0, \theta \geq 0$$

(2) Makeham family:

$$\overline{F_2} (x) = e^{-x - \theta (x + e^{-x} - 1)} \quad , x \geq 0, \theta \geq 0$$

(3) Weibull family:

$$\overline{F_3} (x) = e^{-x^{\theta}} \quad , x \geq 0, \theta \geq 0$$

Note that under $\theta = \theta_0$, the linear failure rate, Makeham and Weibull's distributions reduced to the exponential distribution.
The PAE is defined by:

$$PAE(\delta)=\frac{\left|\frac{d\delta_\theta}{d\theta}\right|_{\theta\to\theta_0}}{\sigma_0},$$

Where

$$\frac{d\delta_\theta}{d\theta}=\frac{\left(6s\mu'_\theta+3s^2\mu'_{2\theta}+s^3\mu'_{3\theta}\phi(s)+s^3\mu_3\,\phi(s)-6\,\phi(s)\right)}{s^3}.$$

In the above three alternatives, we got the following PAEs values:

(i) Linear failure rate family:

$$\text{PAE }(\delta\text{ (s)}) = \frac{s}{(-1-s)^3}$$

(ii) Weibull family:

$$\text{PAE }(\hat{\delta}\text{ (s)}) = \frac{s\,(-0.422784+\log[1-s])}{(-1-s)^2}$$

(iii) Makeham family:

$$\text{PAE }(\hat{\delta}\text{ (s)}) = \frac{s}{(-2+s)\,(-1+s)^2}$$

Table 1 gives the (PAYs) for LFR, Makeham and Weibull families for different values of s.

Table 1. The (PAYs) for LFR, makeham and weibull families

S	**LFR**	**Makehem**	**Weibull**
0.2	4.53313	1.47888	2.8678
0.6	2.41779	0.123295	0.914526
0.7	1.33512	0.168517	0.217619
0.8	0.873926	0.18499	0.010911
0.04	8.30159	2.90418	2.20927

Note that the above results show that these alternatives have a decreasing efficiency in s ≥ 0 at r = 1 and have maximum value at s=0.04

6 Monte Carlo Null Distribution Critical Values

We simulated the null distribution critical points of Monte Carlo for $\hat{\delta}_{(s)}$.

By 10000 simulated sample 5 (5) 50 from the standard exponential distributions.

Table 2. Gives the upper percentile points of the statistic $\hat{\delta}_{(s)}$ with 10000 replications at s = 0.04

N	95%	97%	98%
5	0.667236	0.933895	1.20757
10	0.32912	0.423029	0.511927
15	0.243732	0.308249	0.363627
20	0.202923	0.251669	0.292049
25	0.173466	0.251669	0.292049
30	0.151485	0.184135	0.209312
35	0.133849	0.15805	0.179679
40	0.130626	0.154723	0.173858
45	0.118799	0.139099	0.153607
50	0.111651	0.128273	0.143126

In view of Table 2, it is noticed that the critical values are increasing as the confidence level is increasing and is almost decreasing as the sample size is increasing at s = 0.04 and r = 1

6.1 The power of the proposed test

The power of the proposed test at a significance level α with respect to the alternatives F_1, F_2 and F_3 is calculated and based on simulation data.

We use the significance level α = .05 and the next commonly used alternative distributions in reliability theory:
(i) Linear failure rate family:

$$\overline{F_1}(x) = e^{-x-\frac{\theta}{2}x^2}, \quad x \geq 0, \theta \geq 0$$

(ii) Weibull family:

$$\overline{F_3}(x) = \exp(-x^{\theta} > 0, \theta \geq 0)$$

(iii) Makeham family:

$$\overline{F_4}(x) = e^{-x-\theta(x+e^{-x}-1)}, \quad x \geq 0, \theta \geq 0$$

Based on 10000 replications, Table 3 below shows the power estimate using α = .05 with parameter θ = 1, 2 and 3 at n = 10, 20 and 30.

Table 3. The power estimate for LFR, mekeham and weibull families

N	θ	LFR	Mekeham	Weibull
10	1	0.9996	1.000	0.9553
	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
20	1	1.000	1.000	0.9498
	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
30	1	1.000	1.000	0.961
	2	1.000	1.000	1.000
	3	1.000	1.000	1.000

It is clear from the above table that our test has good powers for all alternatives and the powers increase as the sample size increases. The power is getting as smaller as the *$NBRU_{mgf}$* approaching to the exponential distribution.

7 Testing for Censored Data

In this part, a censor data is the only information available in a life testing model or in a study on producing devices in a factory where some of them may be lost (censored) before the completion of a study.
This experimental situation can formally be modeled as follows:

- We supposed that *n* objects are put on test denote their true life time.
- We let X_1 , X_2 …, X_n to be independent, identically distributed (i.i.d) according to a continuous life distribution F. Let $y_1,y_2\ldots,y_n$ be (i.i.d) according to continuous life distribution G.
- Also we assumed that X's and y's are independent. In the randomly right censored model, we observed the pairs (zj, δ j), j = 1….., n where zj = min (Xj, yj) and

$$\delta_j = \begin{cases} 1, & \text{if } Z_j = X_j \text{ (j} - \text{th observation is uncensored)} \\ 0, & \text{if } Z_j = Y_j \text{ (j} - \text{th observation is uncensored)} \end{cases}$$

Let $Z_{(0)} = 0 < Z_{(1)} < Z_{(2)} < ... < Z_{(n)}$ denote the ordered Z's and $\delta_{(j)}$ is δ_j corresponding to $Z_{(j)}$. Using the censored data (Z_j,δ_j), j = 1, ..., n. Kaplan and Meier [7] proposed the product limit estimator,

$$\overline{F}_n(\chi) = \prod_{[j:z_{(j)}\le X]} \{(n-j)/(n-j+1)\}^{\delta_{(j)}}, X \in [0, Z_{(n)}]$$

Now, for testing $H_0 : \delta(s) = \overline{0}$ against $H_1 : \delta(s) > 0$, using the randomly right censored data, we propose the following test statistic:

$$\hat{\delta}(s) = 6 s\mu + 3s^2 \mu 2 + 6 + (s^3 \mu 3 - 6)\, \phi(s)$$

Where:

$\phi(s) = \int_0^\infty e^{-su}\, dFn(u)$. For computational purposes, (s) may be $\hat{\delta}$.

Rewritten as = $\hat{\delta}_c$ (s) = 6sϕ + 3s2Ω + 6 + (s3A – 6)θ

$$\Phi = \sum_{k=1}^{n} \prod_{m-1}^{k-1} C_m^{\delta(m)} (Z_{(k)} - Z_{(k-1)}),$$

$$\Phi = \sum_{j=1}^{n} e^{-sz(j)} \left[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)} \right],$$

$$\Omega = 2 \sum_{i=1}^{n} \prod_{v=1}^{i-1} C_v^{\delta(v)} (Z_{(i)} - Z_{(i-1)}),$$

And:

$$d F_n (Z_j) = \overline{F_n} (Z_{j-1}) - \overline{F_n} (Z), C_k$$

$= [n - k] [n - k + 1]^{-1}$

Table 4 below gives the critical values percentiles of $\hat{\ }\delta_c$ (s) test for sample sizes n = 5 (5) 30 (10), 81, 86.

Table 4. The upper percentile of ˆδ c(s) with 10000 replications at s=0.04

N	90%	95%	98%
10	2.62874×10^{6}	3.16744×10^{8}	7.89211×10^{10}
15	2.06754×10^{11}	3.02949×10^{14}	6.00685×10^{17}
20	5.32377×10^{15}	3.15229×10^{20}	6.06459×10^{24}
25	2.21669×10^{21}	1.87612×10^{26}	2.65332×10^{31}
30	4.33687×10^{26}	9.53069×10^{32}	1.21239×10^{39}
40	1.27359×10^{37}	5.14191×10^{45}	2.32996×10^{54}
50	5.7179×10^{49}	3.34964×10^{59}	7.10419×10^{69}
60	5.74059×10^{60}	2.70349×10^{72}	8.60736×10^{84}
70	3020586×10^{72}	7.91175×10^{85}	7.96825×10^{102}
81	3.866093×10^{84}	1.40948×10^{101}	3.28718×10^{117}
86	3.91415×10^{91}	5.76665×10^{107}	6.24222×10^{126}

It can be noticed from Table 3 that the critical values are increasing as the confidence level is increasing and are decreasing as the sample size increasing.

7.1 The Power for Censored Data

Based on 10000 replications, Table 5 below shows the power estimate using $\alpha = .05$ with parameter $\theta = 1$, 2 and 3 at n = 10, 20 and 30.

Table 5. The upper percentile of ˆδ c(s) with 10000 replications at s=0.04

N	Θ	LFR	Mekeham	Weibull
10	1	1.000	1.000	0.1
	2	1.000	1.000	0.000
	3	1.000	1.000	0.000
20	1	1.000	1.000	1.000
	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
30	1	1.000	1.000	1.000
	2	1.000	1.000	1.000
	3	1.000	1.000	1.000

8. Applications

In this section, we apply the test on some data-sets to elucidate the applications of the $NBRU_{mgf}$ in both the non censored and the censored data at 95% confidence level.

8.1 Non censored data

Data-set #1

Consider the data-set in Abouammoh et al. [8], these data represent a set of 40 patients suffering from blood cancer (leukemia) from one of ministry of health hospitals in Saudi Arabia. In this case, we get $\delta\hat{\ }(s) = 4.12733$

which is greater than the critical value of the Table 2. Then we accept $H_1$the alternative hypotheses which states that the data set has *NBRUmgf* property and it is not exponential.

Data-set # 2

Consider the data-set which has been given in Grubs [9] and have been used in Ebrahim et al. [10] and Shapiro [11]. This data set gives the times between arrivals of 25 customers at a facility. In this case, we get $\hat{\delta}(s)$ = 8.82008 which is greater than the critical value of the Table 2. Hence we accept H_1 which states that the data set has $NBRU_{mgf}$ property and it is not exponential.

Data-set #3

Consider the data-set which has been given in Lawless [12]. These data-set represent failure times in hours, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress. Here, we get $\hat{\delta}(s)$ = 0.0544243 which is less than the critical value of the Table 2. Hence we accept the null hypothesis.

Data-set #4

Consider the data-set which has been given in Fisher [13] which represent the differences in heights between cross- and self- fertilized plants grown together in one pot. In this case, we get $\hat{\delta}(s) = -27.5858$ which is less than the critical value of the Table 2. Hence we conclude that this data set has exponential distribution.

Data-set #5

Consider the data-set which has been given in Kots and Johnson [14] which represents the survival times (in years) after diagnosis of 43 patients with a certain kind of leukemia. In this case, we get $\hat{\delta}(s)$ = 2.60809 which is greater than the critical value of the Table 2. Hence we accept H1 which states that the data set has $NBRU_{mgf}$ property and is not exponential.

8.2 Censored data

Data-set #6

Consider the data from Susarla and Vanryzin [15], which represents 81 survival times (in months) of patients of melanoma. Out of these 46 represents non-censored data, and the ordered values. Taking into account the whole set of survival data (both censored and uncensored), and computing the statistic from (4) censored data, we get $\hat{\delta}_c(s) = 1.499095993186504 \times 10^{479}$ which is greater than the critical value of the Table 4 at 95% upper percentile. Then, we accept H_1which states that the data set has $NBRU_{mgf}$ property and is not exponential.

Data-set #7

On the basis of right censored data for lung cancer patients from Pena [16], these data consists of 86 survival times (in month) with 22 right censored. Taking into account the whole set of survival data (both censored and uncensored), and computing the statistic from (4) censored data, we get $\hat{\delta}_c(s) = 1.26129 \times 10^{221}$ which is greater than the critical value of the Table 4 at 95% upper percentile. Then, we accept H_1, which states that the data set has $NBRU_{mgf}$ property and is not exponential.

9 Conclusion

A new test for exponentility versus $NBRU_{mgf}$ based on moment inequalities is established. Pitman's asymptotic efficiencies, powers and critical values of the new test are calculated to assess the performance of the test. The right censored data is also handled. Finally some applications are applied to the new test.

Competing Interests

Authors have declared that no competing interests exist.

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